Chapter 15 Integration

0606/22/F/M/19

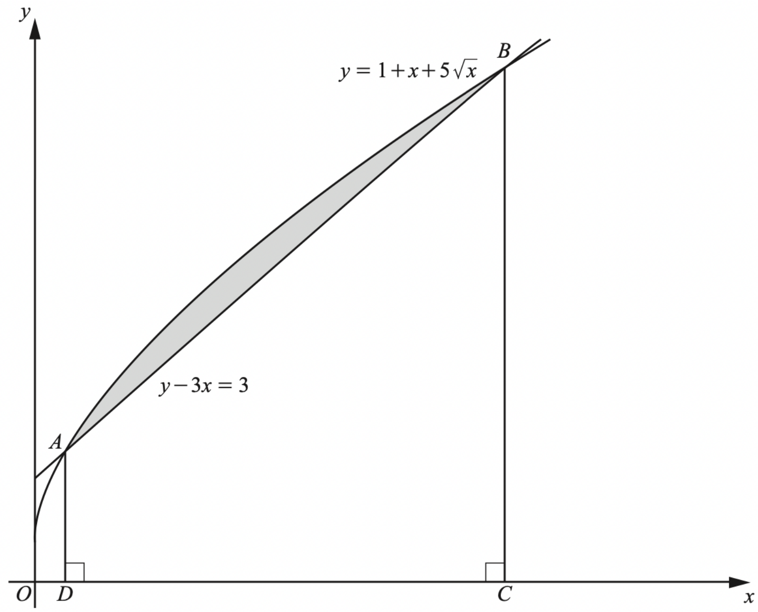
1. A curve is such that when $x = 0$, both $y = -5$ and $\frac{dy}{dx} = 10$. Given that $\frac{d^2y}{dx^2} = 4e^{2x} + 3$, find
    a. The equation of the curve,

[7]

    b. The equation of the normal to the curve at the point where $x = \frac{1}{4}$.

[3]

2.



The diagram shows the curve $y = 1 + x + 5\sqrt{x}$ and the straight line $y - 3x = 3$. The curve and line intersect at the points *A* and *B*. The lines *BC* and *AD* are perpendicular to the *x*-axis.

(i) Using the substitution $u^2 = x$, or otherwise, find the coordinates of *A* and of *B*. You must show all your working.

[6]

(ii) Find the area of the shaded region, showing all your working.

[6]

3. (a) Find $\int \frac{x^2(x^6+1)}{x^6} dx$

[3]

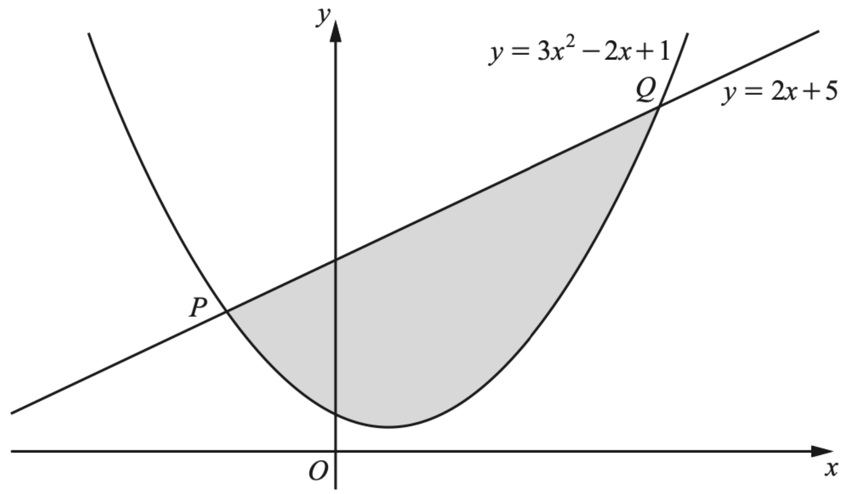
(b) (i) Find $\int cos\,(4\theta\ -\ 5)d\theta$.

[2]

(ii) Hence $\int_{1.25}^{2} cos\,(4\theta\ -\ 5)d\theta$.

[2]

4.



The diagram shows the curve $y = 3x^2 - 2x + 1$ and the straight line $y = 2x + 5$ intersecting at the point P and Q. Show all your workings, find the area of the shaded region.

[8]

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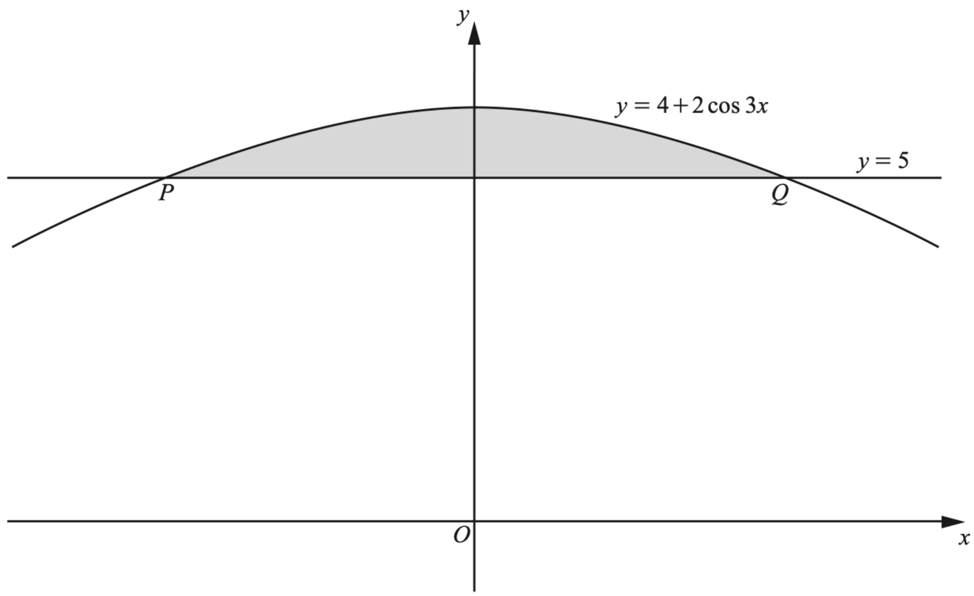
5.



The diagram shows the curve $y = 4 + 2cos\,3x$ intersecting the line y = 5 at the points P and Q.

(i) Find, in terms of r, the x-coordinate of P and of Q.

[3]

(ii) Find the exact area of the shaded region. You must show all your working.

[6]

6. A curve is such that $\frac{d^2y}{dx^2} = (2x + 3)^{-\frac{1}{2}}$. The curve has a gradient of 5 at the point where x = 3 and passes through the point $(\frac{1}{2}, -\frac{1}{3})$.

(i) Find the equation of the curve.

[7]

(ii) Find the equation of the normal to the curve at the point where $x = 3$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[4]

7. (a) Given that $\int_{0}^{a} e^{2x} dx = 50$, find the exact value of *a*. You must show all your working.

[4]

(b) A curve is such that $\frac{dx}{dy} = 3 - 2cos\,5x$. The curve passes through the point $(\frac{\pi}{5}, \frac{8\pi}{5})$.

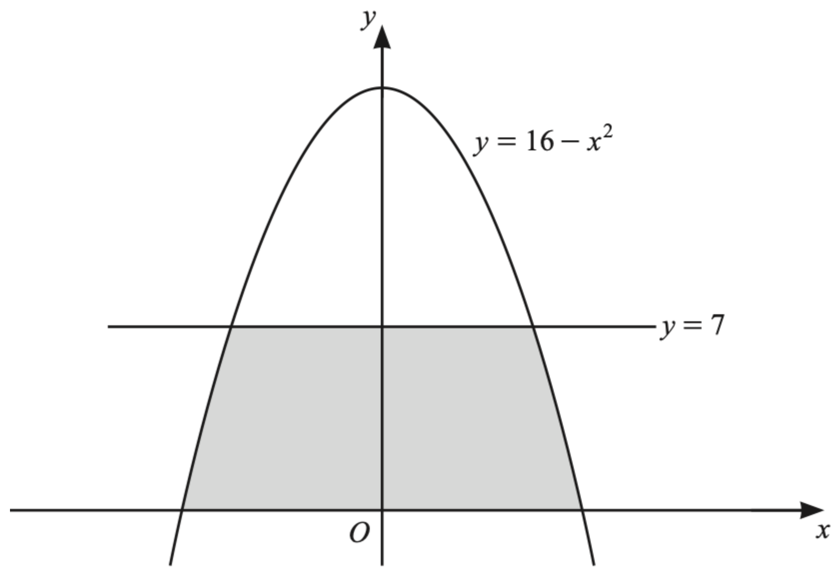
(i) Find the equation of the curve.

[4]

(ii) Find $\int y\,dx$ and hence evaluate $\int_{\frac{\pi}{2}}^{\pi} y\,dx$.

[5]

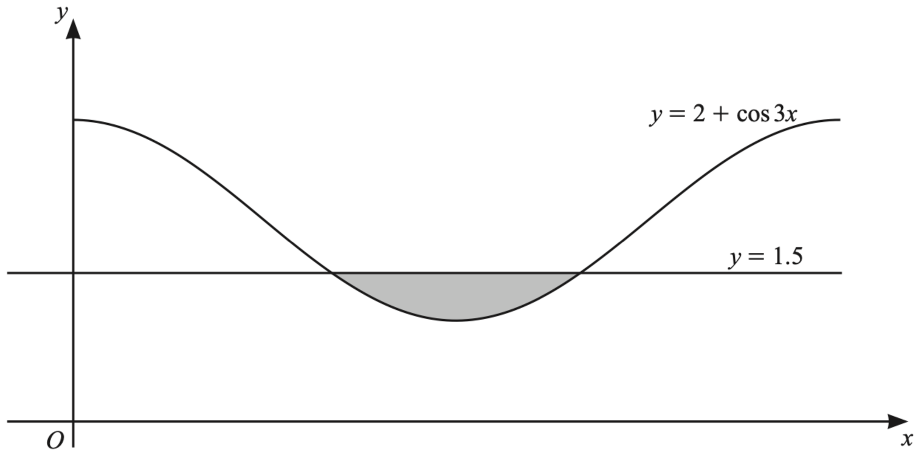
8.



The diagram shows the curve $y = 16 - x^2$ and the straight line y = 7. Find the area of the shaded region. You must show all your working.

[6]

9.



The diagram shows part of the graph of $y = 2 + cos\ 3x$ and the straight line $y = 1.5$. Find the exact area of the shaded region bounded by the curve and the straight line. You must show all your working.

[9]

10. A curve is such that $\frac{d^2y}{dx^2} = 2sin(x + \frac{\pi}{3})$. Given that the curve has a gradient of 5 at the point $(\frac{\pi}{3}, \frac{5\pi}{3})$, find the equation of the curve.

[8]

11. A curve is such that $\frac{d^2y}{dx^2} = 2(3x - 1)^{-\frac{2}{3}}$. Given that the curve has a gradient of 6 at the point $(3, 11)$, find the equation of the curve.

[8]

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12. (i) Differentiate $(x^2 + 3)ln(x^2 + 3)$ with respect to x.

[3]

(ii) Hence find $\int xln(x^2 + 3)dx$.

[2]

13. The equation of a curve is given by $y = xe^{-2x}$.

a. Find $\frac{dy}{dx}$.

[3]

b. Using your answer to **part (i)**, find $\int xe^{-2x} dx$.

[3]

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14. (i) Given that $y = \frac{lnx}{x^2}$, find $\frac{dy}{dx}$.

[3]

(ii) Using your answer to **part (i)**, find $\int \frac{lnx}{x^2} dx$.

[3]

(iii) Hence evaluate $\int_1^2 \frac{lnx}{x^2} dx$.

[2]

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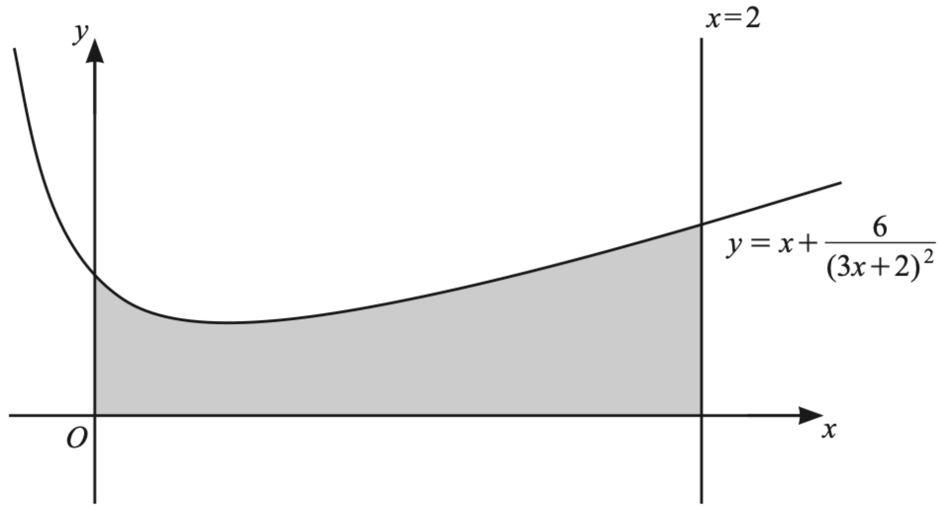
15.



The diagram shows part of the curve $y = x + \frac{6}{(3x+2)^2}$ and the line $x = 2$.

(i) Find, correct to 2 decimal places, the coordinates of the stationary point.

[6]

(ii)Find the area of the shaded region, showing all your working.

[4]